Frequency Domain Properties Verification of Analog Circuits Using SMT

Hafiz ul Asad, Kevin Jones, Frederic Surre

FAC Workshop,UC Berkeley,CA,February 15,2013

CITY UNIVERSITY LONDON

EST 1894

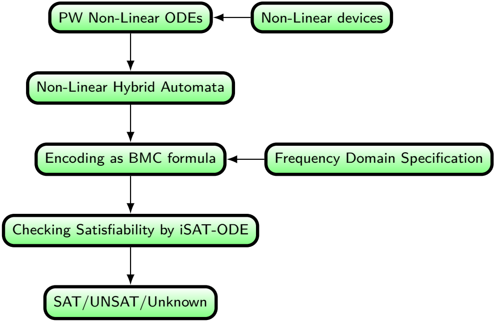
This work is partly sponsored by Mentor Graphics.

- Objective of the Research
- Methodology
- Results
- Conclusion and Future Work

- Are Safety and Stability properties verification of AMS Circuits enough?
- Will Design engineers be not Interested, that for a set of initial conditions and set of parameter variations:
  - An Oscillator circuit does oscillate with a specified frequency, and does not have undesired harmonics.
  - A filter circuit has the desired bandwidth
  - PLL output locks to the input frequency.
- How to do this?
  - Harmonic Balance(Steady state of Non-linear Circuits),Formally possible?
  - Value set of a Transfer function(Computationally prohibitive for Piecewise linear/affine circuits).
  - Unified methodology for Transients and Steady State is difficult.
- Problem Statement
  - Ignoring transients and Using time domain reachability analysis, we verify that for a set of initial conditions,an oscillator circuit does oscillate with a specific fundamental frequency, and does not have undesired harmonics.

CITY UNIVERSITY LONDON

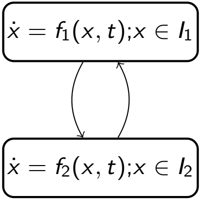
SMT based BMC of Non-Linear Hybrid Automata



- Hybridization
- Using MATLAB/Simulink simulation to compute frequency domain specification
- Target State = The one which violates the desired frequency domain property.
- iSAT-ODE returns an Unknown solution if its deduction algorithm can't establish SAT/UnSAT of a formula.Needs Careful attention.

Hybrid System Verification Using iSAT-ODE

- Extension of iSAT with enclosure methods for ODEs



- Use predicative encoding of hybrid transition system
- iSAT-ODE Formula

$$\Phi = DECL[0] \wedge .. \wedge DECL[k] \wedge Init[0] \wedge Trans[0,1] \wedge .. \wedge Trans[(k-1),k] \wedge Target[k]$$

- Simple Bounds on Variables
- Predicate for Initial Condition
- Transition Relation between Pre and Post valuations of variables
- Target State whose Reachability is to be determined

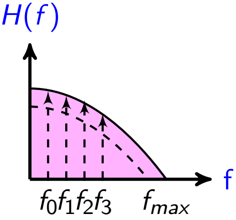
Andreas Eggers et al. SAT modulo ODE: A direct SAT approach to hybrid systems.ATVA 2008

- Representing Periodic Scalar function by Finite Fourier Series,

$$g(t) = a_0 + \sum_{k=1}^{n} (a_k \cos(2\pi f_0 kt) + (b_k \sin(2\pi f_0 kt))$$

- Periodic function membership in power spectral envelop $H(f)$:

  1. $\forall k \in \mathbb{N}, (k.f_0 > f_{max}) \implies a_k = 0$, and $b_k = 0$,
  2. $(a_k^2 + b_k^2) \leq H(k.f_0) \;\; \forall k \in \mathbb{N}$, such that $0 < kf_0 < f_{max}$,
  3. $-H(0) \leq a_0 \leq H(0)$



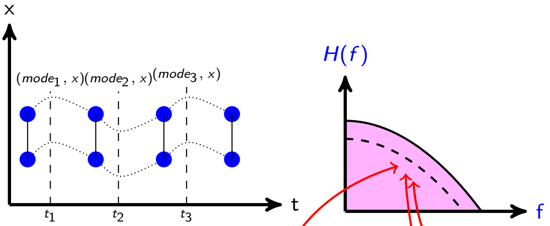
Chakarov, et.al, Combining Time and Frequency Domain Specifications for Periodic Signals, RV 2011.

Robustness of Membership in a Power spectral Envelope

- We are interested in, that a periodic trajectory $g(t)$ is within ϵ degree of robustness of a trajectory $s(t)$ which belongs to power spectral envelop $H(f)$ i.e.,

$$distance(s(t), g(t)) \leq \epsilon$$

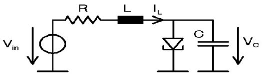
- Extending the idea to hybrid timed traces of oscillator circuit,



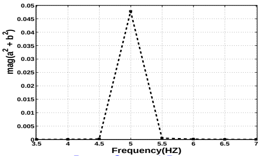
- Membership of Hybrid timed traces to a specified power spectral envelop is ascertained iff,the following Target predicate would make the BMC formula unsatisfiable,

$$\bigwedge_t \Bigg(-\epsilon \geq \Big(((mode_i, x), t_i) - \{a_0 + \sum_{k=1}^{n} (a_k \cos(2\pi f_0 k t_i) + (b_k \sin(2\pi f_0 k t_i))\} \Big) \vee$$

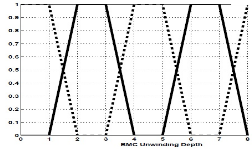
$$\Big(((mode_i, x), t_i) - \{a_0 + \sum_{k=1}^{n} (a_k \cos(2\pi f_0 k t_i) + (b_k \sin(2\pi f_0 k t_i))\} \Big) \geq \epsilon \Bigg)$$



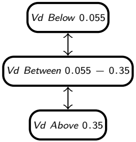
Tunnel Diode Oscillator Circuit Diagram



Power Spectral Density



Oscillating Modes of TDO as Generated by iSAT-ODE

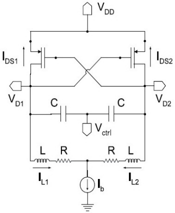


Non-Linear Hybrid Automata Model

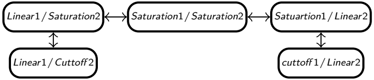
Depth	Decision	Time(Seconds)
0	Unsatisfiable	0
1	Unsatisfiable	81.07
2	Unsatisfiable	83.22
3	Unsatisfiable	304.37
4	Unsatisfiable	352.44
5	Unsatisfiable	1299.64
6	Unsatisfiable	1448.71
7	Unsatisfiable	26779.75
8	Unsatisfiable	27096.21

iSAT-ODE Results

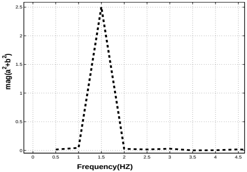
Example2: Voltage Controlled Oscillator



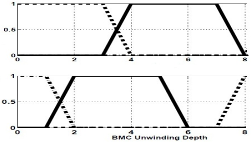
VCO Circuit Diagram



Non-Linear Hybrid Automata Model



Power Spectral Density



Oscillating Modes of VCO as Generated by iSAT-ODE

Depth	Decision	Time(Seconds)
0	Unsatisfiable	0
1	Unsatisfiable	6.13
2	Unsatisfiable	6.26
3	Unsatisfiable	6.73
4	Unsatisfiable	6.79
5	Unsatisfiable	10.58
6	Unsatisfiable	10.68
7	Unsatisfiable	12.34
8	Unsatisfiable	12.51

iSAT-ODE Results

- Given the existing capabilities of SMT solvers,We have formally verified oscillators for a given fourier series based frequency domain property.
- Results are satisfactory and could be extended to more realistic circuits.
- Efficient solvers,would make it possible to verify circuits for deep Unwindings.
- We aim to extend the methodology to reason about transients by introducing fourier transform based specifications
- A unified time and frequency domain methodology, (Different time ranges,Different frequency domain specification using Short time fourier or wavelet transforms).

THANKS