Formal Verification of Continuous Models of Analog Circuits

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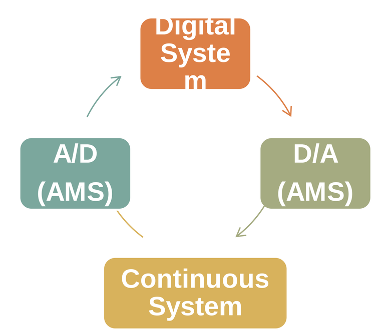
Motivation



AMS Circuits

- Characterized by having both analog and digital circuits
- Interface digital designs to continuous real-world environment
- Integral part of all embedded systems

Functional Verification of AMS Circuits

- Implementation == specification

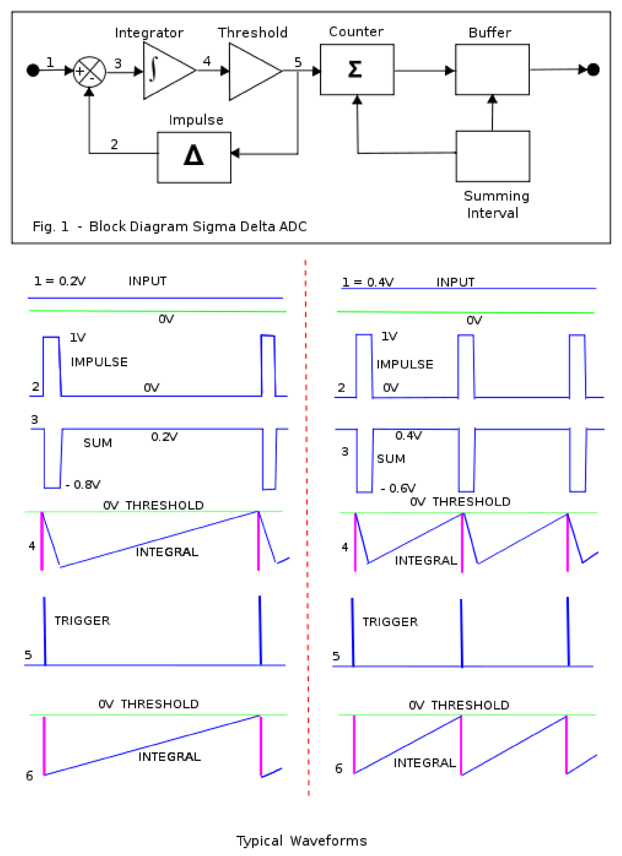
Simulation is the traditional AMS Verification Technique

- Various SPICE Simulators are available
- However cannot guarantee reliable verification results
  - Incomplete verification
  - Inaccurate analysis
  - Time consuming

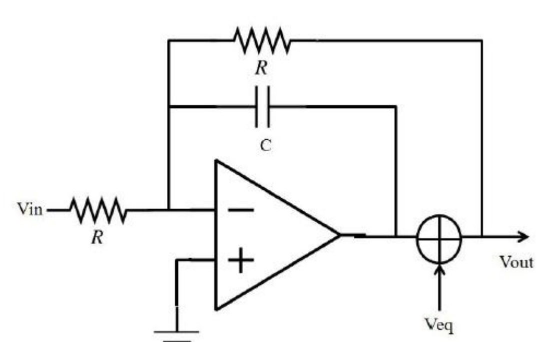
Case Study

Delta Sigma Modulator

Encodes analog signals into digital signals or higher-resolution digital signals into lower-resolution digital signals



First Order Δ-∑ Modulator



Modeling Differential Equation

$$\frac{dv_{out}(t)}{dt} + \frac{1}{RC}v_{out} = \frac{-1}{RC}v_{in} + \frac{dv_{eq}(t)}{dt}$$

Formal Verification Methods

Formal Methods based Techniques are Accurate and Exhaustive

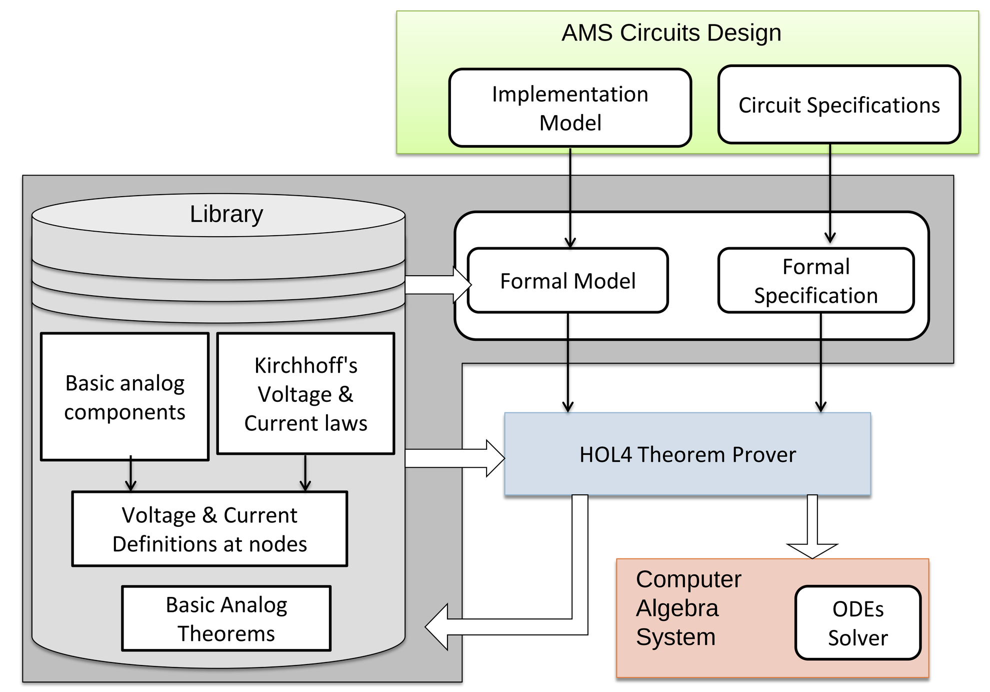
1. Model Checking:

- Discretized models for differential equations
- Computationally expensive
- State-space explosion problem
- Memory limitation

2. Higher-order Logic Theorem Proving

- Framework for continuous analog models
- Complete verification
- Exact solution

Proposed Methodology



1. Implementation model and circuit specifications are translated to HOL4 theorem prover using library.
2. Library mainly consists of analog circuit components definitions and Kirchhoff's Voltage & Current laws.
3. Goal depicting implementation model implies specification is verified in a Theorem Prover.
4. Computer Algebra Software is used for finding the solution of verified differential equations.

Library Definitions

Basic circuit components definition

Definition 1: *Resistor, Inductor, Capacitor and Op-amp*

```
⊢ ∀ R i.resistor_voltage R i = (λt.i t * R)
⊢ ∀ L i.inductor_voltage L i =
    (λt. L * deriv i t)
⊢ ∀ C i Vo. capacitor_voltage C i Vo =
    (λt. Vo + 1/C * integral (0,t) i)
⊢ ∀ Vpos Vneg A. op_amp_voltage Vpos Vneg A=
    (λt. A * (Vpos t - Vneg t))
```

Circuit simplifiers

Definition 2: *Kirchhoff's Current Law*

```
⊢ ∀ V t. kcl I t =
  (∀ x. 0 < x ∧ x < t ⇒
    (sum (0,LENGTH I) (λn. EL n I x) = 0))
```

HOL4 Theorem Prover

Higher-order Logic Theorem Prover

Developed at University of Cambridge

Uses Robin Milner's LCF approach

Core consists of 5 basic axioms and 8 primitive inference rules

Notation : ML

Numerous proof assistants available

Built-in mathematical theories

- Booleans, Sets, Lists, Real Analysis, Natural Numbers, Integers, arithmetic
- Integration, Measure, Probability, Random Variables
- Reliability

Results

Implementation Model of Delta Sigma Modulator

```
⊢ ∀ R C Vin Vout Vc Veq y.
delta_sigma_imp R C Vin Vout Vc Veq y =
  (kcl [resistor_current R Vin;
        resistor_current R Vout;
        capacitor_current C (λx. -Vc x)] t)∧
  (Vout = (λt. Veq t - Vc t))
```

Behavioral Model of Delta Sigma Modulator

```
⊢ ∀ R C Vin Vout Veq y.
delta_sigma_behav R C Vin Vout Veq y =
  (diff_eq [1; R * C] Vout y =
  -Vin y + diff_eq [0; R * C] Veq y)
```

Theorem: Implementation implies specification

```
⊢ ∀ R C Vin Vc Vout Veq t .
  delta_sigma_imp R C Vin Vc Vout Veq t⇒
  delta_sigma_behav R C Vin Vout Veq t
```